FREE SAMPLE · FROM THE INTRO STATS EXAM 1 GUIDE

Three methods, drawn

These are three sections of Part 3 of the full guide, exactly as they appear in it: standard deviation by hand, z-scores, and the 68-95-99.7 rule. The full guide has 8 parts (the topic list is at waar.com.hk/study), a one-page formula sheet, a term drill, a 50-minute mock exam with a worked key, and calculator key presses.

3.2 Spread: range, variance and standard deviation

The **range** is max − min: quick, but two values decide it. The **standard deviation (s)** uses every value: it is, roughly, the typical distance a value sits from the mean.

1 ▶ *Find the balance point: add the values, divide by n.*
the term: **mean $\bar{x}$**

2 ▶ *Measure each value's distance from the mean, keeping the sign (left = negative).*
the term: **deviation $(x - \bar{x})$**

3 ▶ *Square each distance so the negatives can't cancel the positives (the deviations always sum to 0).*
the term: **squared deviation $(x - \bar{x})^2$**

4 ▶ *Add the squares up.*
the term: **sum of squares $\Sigma(x - \bar{x})^2$**

5 ▶ *Divide by n − 1 (one fewer than the count), not n.*
the term: **variance s^2**

6 ▶ *Take the square root, which brings you back to the original units.*
the term: **standard deviation $s = \sqrt{s^2}$**

Worked example. Six quiz scores: 4, 8, 6, 5, 3, 10.

Mean: (4 + 8 + 6 + 5 + 3 + 10) ÷ 6 = 36 ÷ 6 = **6**

x	$x - \bar{x}$	$(x - \bar{x})^2$
4	-2	4
8	2	4
6	0	0
5	-1	1
3	-3	9
10	4	16
sum	0	**34**

> Variance $s^2 = 34 \div (6 - 1) = 34 \div 5 =$ **6.8**. Standard deviation $s = \sqrt{6.8} =$ **2.608**. Read it: a typical score sits about 2.6 points from the mean of 6.

> **Exam trap.** Three things the exam checks: (1) the deviations column sums to **0**, so if yours doesn't, the mean is wrong; (2) divide by **n − 1** for a sample (your calculator calls this Sx; σx divides by n and is for a whole population); (3) s is **never negative** and is 0 only when every value is identical.

Properties of s: same units as the data; bigger spread → bigger s; one extreme outlier inflates it a lot (it is squared); adding a constant to every value leaves s unchanged, multiplying every value by c multiplies s by |c|.

> **Picture it like this:** the mean is the center of town and s is the typical commute: a small s means everyone lives close in, a big s means people are spread out along the highway.

3.5 z-scores: comparing apples with oranges

A **z-score** says how many standard deviations a value sits above (+) or below (−) its mean. Two scores from different scales become comparable once each is turned into "how unusual, in SD units".

1 ▶ *Subtract the mean: how far is the value from the center, with sign?*
the term: **$x - \mu$ (or $x - \bar{x}$)**

2 ▶ *Divide by the standard deviation: how many SD-steps is that?*
the term: **$z = (x - \mu) / \sigma$**

3 ▶ *Read it: z = +1.5 means 1.5 SDs above average; z = −2 means 2 SDs below. Beyond ±2 is unusual, beyond ±3 is rare.*
the term: **standardized value**

Worked example. Alex scored 82 on a test with mean 74 and SD 8. Sam scored 68 on a different test with mean 60 and SD 5. Who did better relative to their class?

Alex: z = (82 − 74) / 8 = 8 / 8 = **1**. Sam: z = (68 − 60) / 5 = 8 / 5 = **1.6**.

> Sam did better relative to the class (1.6 SDs above the mean against Alex's 1.0), even though 68 is a lower raw score.

Going backwards: **x = μ + zσ**. "What score is 1.5 SDs above a mean of 74 with SD 8?" → 74 + 1.5 × 8 = 86.

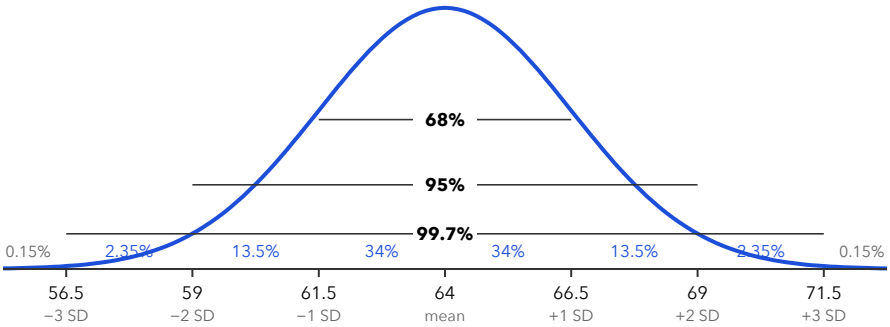
Exam trap. Keep the sign. A negative z is below the mean; the exam loves "which value is more unusual" where the answer is the *larger absolute* z, even if it is negative.

Picture it like this: z is a ruler whose inch is one standard deviation. Every dataset gets measured with its own ruler, so the readings can be compared.

3.6 The empirical rule (68 - 95 - 99.7)

For data whose histogram is **bell-shaped and symmetric** (a normal distribution), three facts hold:

1. ▶ *Walk one SD out from the mean on each side: about 68% of the values live in that band.*
   the term: **μ ± 1σ holds ≈ 68%**
2. ▶ *Walk two SDs out: about 95%.*
   the term: **μ ± 2σ holds ≈ 95%**
3. ▶ *Walk three SDs out: about 99.7%, nearly everything.*
   the term: **μ ± 3σ holds ≈ 99.7%**
4. ▶ *Need a one-sided or in-between piece? Split with symmetry: each tail beyond 2 SD is (100 − 95) / 2 = 2.5%; between 1 and 2 SD on one side is (95 − 68) / 2 = 13.5%.*
   the term: **the slices: 34, 13.5, 2.35, 0.15 (each side)**



Adult women's heights, roughly bell-shaped: mean 64 in, SD 2.5 in. Label the axis with the real values, then read the slices.

Worked example. **Heights: mean 64 in, SD 2.5 in, bell-shaped. (a) Between what heights are the middle 95%? (b) What percent are taller than 69 in? (c) What percent are between 61.5 and 69 in?**

> (a) 64 ± 2 × 2.5 → **59 to 69 in**. (b) 69 is +2 SD; above it is one tail of the 95% band: (100 − 95) / 2 = **2.5%**. (c) 61.5 is −1 SD, 69 is +2 SD: 34% + 34% + 13.5% = **81.5%**.

Exam trap. The rule needs a **bell shape**. For skewed data it is wrong; for "any shape at all" courses sometimes give Chebyshev's rule: at least $1 - 1/k^2$ of the data lies within k SDs (at least 75% within 2 SDs, 89% within 3). If your course didn't mention Chebyshev, skip it.

Picture it like this: a bell with three nested fences. The inner fence (±1 SD) pens 68 of every 100 sheep, the next (±2) pens 95, the outer (±3) pens 997 of 1,000. Each question is "which pens, and which halves of them?"

The full guide: waar.com.hk/study · US$15 · © 2026 W A A R LIMITED, Hong Kong. Original work, not tied to any school or professor.